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CORRECTIONS TO "SUB-RIEMANNIAN GEOMETRY"

ROBERT S. STRICHARTZ

An error in the proof of Corollary 6.2 of [1] has been pointed out by Gerard Ben-Arous. The computation of $M(x, \lambda)$ in the case $\lambda_0 = 0$ on p. 243 is incorrect, because $M(x, \lambda) = 0$ when $\lambda_0 = 0$ and $\lambda_j g^{jk}(x) = 0$ for all k. (There is also a factor of $\frac{1}{2}$ missing in the formula as stated for $\lambda_0 \neq 0$, but this is not significant.) Thus when applying the Pontryagin Theorem (Theorem 6.1) it is necessary to consider one additional case: $\lambda_0 = 0$ and $M(x, \lambda) \equiv 0$ along the curve. The conditions of the theorem in this case lead to the equations

$$\lambda_j g^{jk}(x) = 0, \tag{1}$$

$$\dot{\lambda}_j = -\lambda_p \frac{\partial g^{pq}}{\partial x^j} \xi_q. \tag{2}$$

Now if we differentiate (1), raise indices in (2), and combine, the result is

$$g^{jk} \frac{\partial g^{pq}}{\partial x^j} \lambda_p \xi_q = g^{jq} \frac{\partial g^{pk}}{\partial x^j} \lambda_p \xi_q,$$

which is equivalent to $\Gamma(\xi, \lambda) = 0$.

If we assume the strong bracket generating hypothesis, then Theorem 2.4 implies a contradiction, so this case cannot arise and the proof is complete. However, without the strong bracket generating hypothesis there appears to be nothing to rule out this case, and the Pontryagin theorem gives little useful information. Therefore it appears unlikely that this method of proof can be made to work in the general case.

The addition of the strong bracket generating hypothesis to Corollary 6.2 means that it also has to be added as a hypothesis to Theorems 6.3, 6.4, 7.1(a), 8.2(b), 8.7 and the converse in 9.2. It is already a hypothesis in a number of other theorems.

We also note that the definition of the raised Christoffel symbols (p. 226, formula (2.2)) is exactly the negative of what it should be:

$$\Gamma^{kpq} = -\frac{1}{2}\left(g^{jp} \frac{\partial g^{kq}}{\partial x^j} + g^{jq} \frac{\partial g^{kp}}{\partial x^j} - g^{jk} \frac{\partial g^{pq}}{\partial x^j} \right).$$

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To incorporate this correction into the proofs of Lemmas 2.3 and 2.4 add the minus sign to the right side of the equations on p. 227, l.3 and p. 228, l.2. The correct form of Γ^{kpq} is in fact already used in Lemma 2.6 and §§4 and 5.

We also take this opportunity to correct some typographical errors:

p. 226, l.-5: Read "v_q" for "v_p".

p. 227, l.8: Last equation should read

$$\text{“} = g^{pq}(x)\frac{\partial \psi^k(x)}{\partial x^p}\frac{\partial \psi^j(x)}{\partial x^q}\text{”}.$$

p. 234—In the three-line displayed equation there should be a plus sign before the double summation in the first line and a minus sign after the equal sign in the third line.

p. 237, l.-8: Should read

$$\dot{\xi}_j + \frac{1}{2}\frac{\partial g^{pq}(x)}{\partial x^j}\xi_p\xi_q = \dot{\eta}_j + \frac{1}{2}\frac{\partial g^{pq}(x)}{\partial x^j}\eta_p\eta_q + \left(\dot{v}_j + \frac{1}{2}\frac{\partial g^{pq}(x)}{\partial x^j}\eta_p v_q\right).$$

p. 237, l.-4: Read "$(\dot{\xi}_j + \cdots$" for "$(\xi_j + \cdots$".

p. 240, bottom line: Read

$$\text{“}2\frac{\partial g^{\alpha c}}{\partial x^a}\cdots\text{”} \quad \text{for} \quad \text{“}2\frac{\partial x^{\alpha c}}{\partial x^a}\cdots\text{”}.$$

p. 241, l.5: The minus sign should be replaced by an equal sign.

p. 241, l.-7: Read "generating" for "geometry".

p. 243, l.-12: Insert an equal sign after $-\dot{\lambda}_j$.

p. 247, l.-7: Read "$\psi_j(P)$" for "$\psi(P_j)$".

p. 260, l.4: Read "$=$" for "$\neq$".

Reference

[1] R. S. Strichartz, *Sub-Riemannian geometry*, Journal of Differential Geometry **24** (1986), 221–263.

CORNELL UNIVERSITY